MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2019

11-01-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of 90 questions. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Mathematics, Chemistry and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above.

PART-A-MATHEMATICS

1. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more than one solution, then

(1) $b - c + a = 0$ (2*) $b - c - a = 0$ (3) $a + b + c = 0$ (4) $b + c - a = 0$

Sol. The system of equations

$2x + 2y + 3z = a$ .... (1)

$3x - y + 5z = b$ .... (2)

$x - 3y + 2z = c$ .... (3)

have more than one solution

$\Rightarrow$ (1) + (3) = (2)

$\Rightarrow b = a + c$

2. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to

(1*) $\frac{1}{12}$ (2) $\frac{1}{4}$ (3) $\frac{-1}{12}$ (4) $\frac{5}{12}$

Sol. $F_4(x) = \frac{\sin^4 x + \cos^4 x}{4} = \frac{1 - 2\sin^2 x, \cos^2 x}{4} = \frac{1}{4} - \frac{1}{2}\sin^2 x$

$F_6(x) = \frac{\sin^6 x + \cos^6 x}{6} = \frac{1 - 3\sin^2 x, \cos^2 x(\sin^2 x + \cos^2 x)}{6}$

$= \frac{1}{6} - \frac{1}{2}\sin^2 x.\cos^2 x$

$F_4(x) - f_0(x) = \frac{1}{4} - \frac{1}{6} = \frac{6-4}{24} = \frac{2}{24} = \frac{1}{12}$

3. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is:

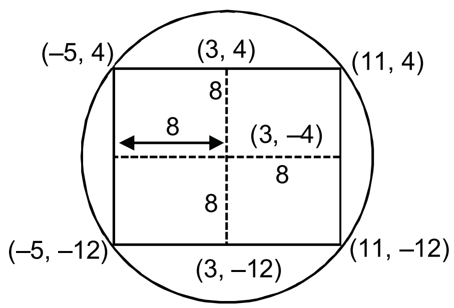
(1) 6 (2) $\sqrt{137}$ (3*) $\sqrt{41}$ (4) 13

Sol. Centre (3, –4)

Radius $= \sqrt{9 + 16 + 103} = \sqrt{128} = 8\sqrt{2}$

$\therefore$ (–5, 4) will be nearer to the (0, 0)

$\therefore \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$



4. If q is false p ^ q ↔ is true, then which one of the following statements is a tautology?

(1) (pr) → (p ^ r) (2*) (p ^ r) → (p r) (3) p ^ r (4) pr

Sol. As $(p \wedge q) \leftrightarrow r$ is true

$\therefore$ If $p \wedge q$ is true and r is true. Or $p \wedge q$ is F and r is F.

As q is false

$p \wedge q$ can not be true

This case is not possible

5. The area (in s units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(1) $\frac{5}{4}$ (2*) $\frac{9}{8}$ (3) $\frac{7}{8}$ (4) $\frac{3}{4}$

Sol. $C_1 : x^2 = 4y$ and $C_2 : x = 4y - 2$

$x^2 = x + 2 \Rightarrow x = 2$ & $x = -1$

Required area $= \frac{1}{4}\left|\int_{-1}^{2}(x^2 - x - 2)dx\right|$

$= \frac{1}{4}\left|\frac{8}{3} - 2 - 4 + \frac{1}{3} + \frac{1}{2} - 2\right| = \frac{9}{8}.$

6. The value of the integral

(where [x] denotes the greatest integer less than or equal to x) is:

(1*) 0 (2) sin 4 (3) 4 (4) 4 – sin 4

Sol. $I = \int_{-2}^{2} \frac{\sin^2 x\, dx}{\left[\frac{x}{\pi}\right] + \frac{1}{2}}$

$I = \int_{-2}^{2} \frac{\sin^2 x\, dx}{-\left[\frac{x}{\pi}\right] - \frac{1}{2}} \Rightarrow 2I = 0 \Rightarrow I = 0$

7. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then | d | equals

(1) $\frac{2}{3}$ (2) 2 (3) $\frac{\sqrt{5}}{2}$ (4*) $\sqrt{2}$

Sol. Variance remains some if same number is subtracted from each observation. (subtract 10 from each observation)

$\therefore \frac{10(-d)^2 + 10(0)^2 + 10(d)^2}{30} - \left(\frac{10(-d) + 10(0) + 10(d)}{30}\right)^2 = \frac{4}{3}$

$\frac{20d^2}{30} = \frac{4}{3}$

$\Rightarrow d^2 = 2$

$(d) = \sqrt{2}$

8. The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane 2x + 3y – z = 5, contains which one of the following points?

(1) (2, 2, 0) (2) (–2, 2, 2) (3) (0, –2, 2) (4*) (2, 0, –2)

Sol. Equation of line passing through (3, – 2, 1) and perpendicular to the plane 2x + 3y – z = 5 is

$\frac{x-3}{2}=\frac{y+2}{3}=\frac{z-1}{-1}$ ...... (1)

Let point of intersection of (1) and the plane is

P(2λ + 3, 3λ – 2, –λ + 1) then

$$4\lambda + 6 + 9\lambda - 6 + \lambda - 1 = 5 \Rightarrow \lambda = \frac{3}{7}$$

$P\left(\frac{27}{7}, \frac{-5}{7}, \frac{4}{7}\right)$

Required plane is $\begin{vmatrix} x-3 & y+2 & z-1 \\ 2 & -1 & 3 \\ 6 & 9 & -3 \end{vmatrix} = 0$

$\Rightarrow -24(x-3) + 24(y+2) + 24(z-1) = 0$

$\Rightarrow -x + y + z = -4$

9. Two integers are selected at random from the set [1, 2, ..., 11]. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:

(1) $\frac{7}{10}$ (2) $\frac{1}{2}$ (3*) $\frac{2}{5}$ (4) $\frac{3}{5}$

Sol. Number of ways to select 2 even number = ${}^5C_2 = 10$

Number of ways to select 2 odd number = ${}^6C_2 = 15$

Required probability = $\frac{10}{25}=\frac{2}{5}$

10. Two sides with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of theses circles is:

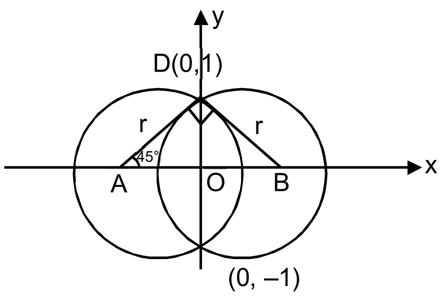
(1) 1 (2*) 2 (3) $2\sqrt{2}$ (4) $\sqrt{2}$

Sol. The two circle will be orthogonalx

OD = 1

∴ OA = OB = OD = 1

⇒ AB = 2



11. The value of r for which ${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots\ldots\ldots\ldots + {}^{20}C_0\,{}^{20}C_r$ is maximum, is:

(1) 15 (2*) 20 (3) 11 (4) 10

Sol. ${}^{20}C_r, {}^{20}C_0 + {}^{20}C_{r-1}, {}^{20}C_1 + \ldots\ldots\ldots {}^{20}C_0 . {}^{20}C_r =$

Selecting r student from 20 boys and 20 girls $^{40}C_r$

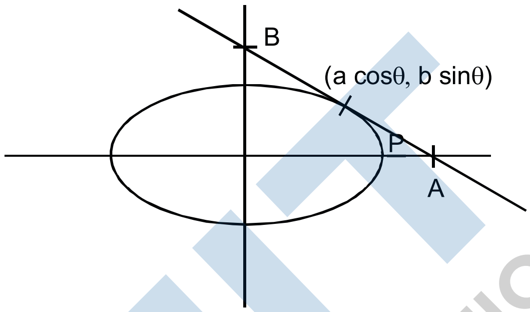
$^{40}C_r$ will be maximum if r = 20.

12. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinates axes lie on the curve:

(1) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (2) $\frac{x^2}{4}+\frac{y^2}{2}=1$ (3*) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (4) $\frac{x^2}{2}+\frac{y^2}{4}=1$

Sol. Equation of tangent is

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$



A is $\left(\frac{a}{\cos\theta}, \theta\right)$

B is $\left(0, \frac{b}{\sin\theta}\right)$

Let P(h, k) is mid point

$2h=\frac{a}{\cos\theta}$

$2k=\frac{b}{\sin\theta}$

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \frac{a^2}{4h^2}+\frac{b^2}{4k^2}=1$

$\Rightarrow \frac{2}{4x^2}+\frac{1}{4y^2}=1$

$\Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2}=1$

13. Let $f(x) = \begin{cases} -1, & -2\le x<0 \\ x^2-1, & 0\le x\le 2\end{cases}$ and g(x) = |f(x)| + f(|x|). Then, in the interval (–2, 2), g is

(1) differentiable at all points (2) not continuous

(3) not differentiable at two points (3*) not differentiable at one point

Sol. $g(x) = |f(x)| + f(|x|) = \begin{bmatrix} x^2 & -2\le x<0 \\ 0 & 0\le x<1 \\ 2(x^2-1) & 1\le x\le 2\end{bmatrix}$

$\Rightarrow$ g (x) is not differentiable at x = 1

14. If $x \log_e(\log_e x) - x^2 + y^2 = 4$ (y > 0), then $\frac{dy}{dx}$ at x = e is equal to

(1) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (2*) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (3) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (4) $\frac{e}{\sqrt{4+e^2}}$

Sol. $y' = \dfrac{-\left(\ell n \ell n x + \dfrac{1}{\ell n x} - 2x\right)}{2y}$

$\Rightarrow y(e) = \sqrt{4+e^2}$

$\Rightarrow y'(e) = \dfrac{2e-1}{2\sqrt{4+e^2}}$

15. If $\int \dfrac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$ for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals:

(1) $\dfrac{-1}{27x^9}$ (2*) $\dfrac{-1}{3x^3}$ (3) $\dfrac{1}{27x^6}$ (4) $\dfrac{1}{9x^4}$

Sol. $I = \int \dfrac{1}{x^4}\sqrt{1-x^2}\,dx = \int \dfrac{1}{x^3}\sqrt{\dfrac{1}{x^2}-1}\,dx$

$\dfrac{1}{x^2} - 1 = t^2 \Rightarrow \dfrac{dx}{x^3} = -tdt$

$I = \int -t^2 dt = \dfrac{-\left(\sqrt{1-x^2}\right)^3}{3x^3} + C$

$\Rightarrow A(x) = -\dfrac{1}{3x^3}$ and $m = 3$

16. Let [x] denote the greatest integer less than or equal to x. Then $\lim\limits_{x\to 0}\dfrac{\tan\left(\pi\sin^2 x\right) + \left(|x| - \sin(x[x])\right)^2}{x^2}$

(1*) does not exist (2) equal π (3) equals $\pi + 1$ (4) equals 0

Sol. $LHL = \lim\limits_{x\to 0}\dfrac{\tan(\pi\sin^2 x)}{x^2} + \left(1 + \dfrac{\sin x}{-x}\right)^2 = \pi$

$RHL = \lim\limits_{x\to 0}\dfrac{\tan(\pi\sin^2 x)}{x^2} + 1 = \pi + 1$

$LHL \neq RHL$

17. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in \mathbf{R} : x^2 + 30 \leq 11x\}$ is:

(1) –122 (2) –222 (3*) 122 (4) 222

Sol. $f(x) = 3x(x-3)^2 - 40$

$x^2 + 30 - 11x \leq 0$

$\Rightarrow 5 \leq x \leq 6$

$\Rightarrow f(x) \leq 122$

18. Let $f : R \to R$ be defined by $f(x) = \dfrac{x}{1+x^2}$, $x \in R$. Then the range of *f* is:

(1*) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (2) R – [–1, 1] (3) R $-\left[-\frac{1}{2},\frac{1}{2}\right]$ (4) (–1, 1) – {0}

Sol. $1 + x^2 \geq 2\,|x|$ (AM ≥ GM)

$\Rightarrow \frac{|x|}{1+x^2} \leq \frac{1}{2} \Rightarrow -\frac{1}{2} \leq \frac{x}{1+x^2} \leq \frac{1}{2}$

19. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is

(1) $\frac{1}{3}$ (2*) $\frac{2}{3}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

Sol. $\frac{a}{1-r} = 3$

Cube both sides

$\frac{a}{(1-r)^3} = 27$ ....(1)

and $\frac{a^3}{1-r^3} = \frac{27}{19}$ ....(2)

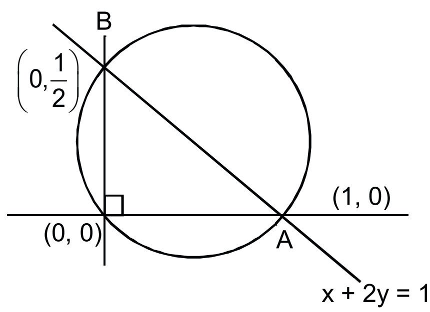
$(1)/(2)$ gives $\frac{1-r^3}{(1-r)^3} = 19$

$r = \frac{2}{3}$

20. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of the perpendicular distances from A and B on the tangent to the circle at the origin is

(1) $\frac{\sqrt{5}}{2}$ (2) $2\sqrt{5}$ (3*) $\frac{\sqrt{5}}{4}$ (4) $4\sqrt{5}$

Sol. Equation of circle $x(x-1) + \left(y - \frac{1}{2}\right)y = 0$



$x^2 + y^2 - x - \frac{y}{2} = 0$

Equation of tangent at (0, 0)

$x.0 + y.0 - \frac{x+0}{2} - \frac{y+0}{2\times 2} = 0$

2x + y = 0 ....(1)

Sum of distance of A and B form Line (i) is

$\frac{2}{\sqrt{5}} + \frac{\frac{1}{2}}{\sqrt{5}} = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$

21. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is

(1) $\frac{3}{2}y$ (2*) $\frac{c}{\sqrt{3}}$ (3) $\frac{c}{3}$ (4) $\frac{y}{\sqrt{3}}$

Sol. $x^2 - c^2 = y$

$(a + b)^2 - c^2 = ab$

$a^2 + b^2 - c^2 = -ab$

$\frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2}$

$\cos c = -\frac{1}{2}$

$c = \frac{2\pi}{3}$

$\sin C = \frac{\sqrt{3}}{2}$

$\frac{c}{\sin c} = 2R \Rightarrow R = \frac{c}{2\sin c} = \frac{c}{\sqrt{3}}$

22. Equation of a common tangent to the parabola $y^2 = 4x$ and the parabola $xy = 2$ is

(1) $x + y + 1 = 0$ (2*) $x - 2y + 4 = 0$ (3*) $x + 2y + 4 = 0$ (4) $4x + 2y + 1 = 0$

Sol. Let the tangent be $y = mx + \frac{1}{m}$

then it must be tangent of $xy = 2$

$\Rightarrow mx^2 + \frac{x}{m} - 2 = 0 \Rightarrow m^2x^2 + x - 2m = 0$ has equal roots

$\Rightarrow 1 - 8m^3 = 0 \Rightarrow m = \frac{1}{2}$

23. The sum of the real values of x for which the middle terms in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is

(1*) 0 (2) 6 (3) 4 (4) 8

Sol. 5^{th} term will be the middle term.

$t_{4+1} = {}^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4 = 5670$

$= {}^8C_4.x^8 = 5670$

$= \frac{8\times7\times6\times5}{4\times3\times2}x^8 = 5670$

$= x^8 = \frac{567}{7} = 81$

$= x^8 - 81 = 0$

$\Rightarrow$ Real value of $x = \pm\sqrt{3}$

24. Let $a_1, a_2, \ldots\ldots\ldots, a_{10}$ be a G.P. $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

(1*) 5^4 (2) $4(5^2)$ (3) 5^3 (4) $2(5^2)$

Sol. $\frac{a_3}{a_1} = \frac{a_1 r^2}{a_1} = r^2$

$\Rightarrow r^2 = 25$

Now $\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = (25)^2 = 5^4$

25. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then |p| is:

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{1}{\sqrt{3}}$ (3*) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{6}}$

Sol. $AA^T = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}\begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow 4q^2 + r^2 = 1;\ r^2 = 2q^2;\ p^2 + q^2 + r^2 = 1$

$\Rightarrow q^2 = \frac{1}{6},\ r^2 = \frac{1}{3} \Rightarrow p^2 = \frac{1}{2} \Rightarrow |p| = \frac{1}{\sqrt{2}}$

26. If y(x) is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$, $x > 0$,

where $y(1) = \frac{1}{2}e^{-2}$, then:

(1) $y(\log_e 2) = \log_e 4$ (2) $y(\log_e 2) = \frac{\log_e 2}{4}$

(3*) y (x) is decreasing in $\left(\frac{1}{2}, 1\right)$ (4) y (x) is decreasing in (0, 1)

Sol. I.F. $= e^{\int\left(2+\frac{1}{x}\right)dx} = xe^{2x}$

solution is given by

$y\,xe^{2x} = \int x\,dx + c \Rightarrow yxe^{2x} = \frac{x^2}{2} + c$

$y(1) = \frac{1}{2}e^{-2} \Rightarrow c = 0$

$y = \frac{xe^{-2x}}{2}$

$y' = \frac{e^{-2x}}{2}(1 - 2x) < 0\ \forall x > \frac{1}{2}$

27. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0 are:

(1) 2, –1, 1 (2) $\sqrt{2}, 2, -\sqrt{2}$ (3*) $\sqrt{2}$, 1, – 1 (4) $2\sqrt{3}$, 1, – 1

Sol. Let the direction cosines be (a, b, c)

then b + c = 0

$b - c = 1 \Rightarrow b = \frac{1}{2}, c = -\frac{1}{2}$

$a = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow$ direction ratios be $(\pm\sqrt{2}, 1, -1)$

28. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is

(1) $-10\hat{i} - 5\hat{j}$ (2) $-14\hat{i} - 5\hat{j}$ (3) $-14\hat{i} + 5\hat{j}$ (4*) $-10\hat{i} + 5\hat{j}$

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$

$\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$

coplaner $\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$

$\Rightarrow (\lambda^3 - \lambda - 16) - 2(\lambda^2 - 1 - 8) + 4(4 - 2\lambda) = 0$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow (\lambda - 2)(\lambda^2 - 9) = 0$

$\Rightarrow \lambda = 2, 3, -3$

$\vec{a} \times \vec{b} = \vec{0}$ or $= \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$

29. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ cube of the other root, then a value of k is

(1) –81 (2) 100 (3) 144 (4*) –300

Sol. $\alpha + \alpha^3 = -\frac{K}{81}$ ....(1)

$\alpha^4 = \frac{256}{81}$

$\alpha = \pm\frac{4}{3}$ ....(2)

Form (1) and (2)

$\frac{4}{3} + \frac{64}{27} = \frac{-K}{81}$

K = – 300

30. Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ $\left(i=\sqrt{-1}\right)$, where x and y are real numbers, then y – x equals

(1*) 91 (2) –85 (3) 85 (4) –91

Sol. $\frac{x+iy}{27} = -8+\frac{2}{3}-4\hat{i}+\frac{1}{27}\hat{i}$

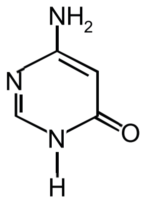
$x + iy = -216 + 18 - 108\hat{i} + \hat{i}$

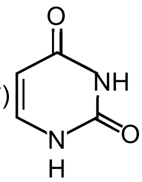
$= \; = -198 - 107\hat{i}$

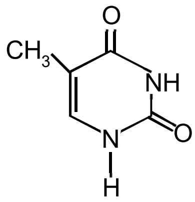
$y - x = 91$

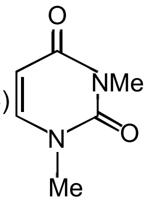
PART-B-CHEMISTRY

31. Among the following compounds, which one is found in RNA?

(1) 

(2*) 

(3) 

(4) 

Sol. Fact based

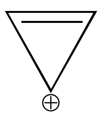
32. The correct match between items I and II is:

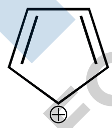
Item–I	**Item:II**
(Mixture)	(Separation method)
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

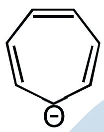
(1) (A)→(S); (B)→(R); (C)→(P)
(2*) (A)→(Q); (B)→(R); (C)→(S)
(3) (A)→(R); (B)→(P); (C)→(S)
(4) (A)→(Q); (B)→(R); (C)→(P)

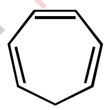
Sol. Fact based

33. Which compound (s) out of the following is(are) not aromatic?

(1) 

(2) 

(3) 

(4) 

(1) (B), (C) and (D)
(2*) (C) and (D)
(3) (B)
(4) (A) and (C)

Sol. b → antiaromatic

d → Non aromatic (no cyclic conjugation)

34. A solid having density of 9×10^3 kg m^{-3} forms face centred cubic crystals of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid?

[Avogadro constant 6×10^{23} mol^{-1}, π 3]

(1) 0.0432 kg mol^{-1}
(2) 0.0216 kg mol^{-1}
(3*) 0.0305 kg mol^{-1}
(4) 0.4320 kg mol^{-1}

Sol. $\text{Density} = \dfrac{Z \times M}{N_{AV} \times a^3}$

Here Z = 4, $a = 200\sqrt{2}$

$N_{AV} = 6.02 \times 10^{23}$, $\quad d = 9 \times 10^3 \dfrac{Kg}{m^3}$

35. For the cell Zn (s)|Zn^{2+} (aq) || M^{x+} (aq) |M(s), different half cells and their standard electrode potentials are given below:

$M^{x+}(aq)/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^{\circ}_{M^{x+}/M}/(V)$	1.40	0.80	0.77	–0.44

If $= E^{\circ}_{Zn^{2+}/Zn}$ 0.76 V, which cathode will give a maximum value of E°_{Cell} per electron transferred?

(1) Ag^{+}/Ag (2) Fe^{3+}/Fe^{2+} (3*) Au^{3+}/Au (4) Fe^{2+}/Fe

Sol. $E^{o}_{cell} = E_{cathode} - E^{o}_{Zn^{2+}/Zn}$

Since $Au^{3+} \rightarrow Au$ has maximum value.

36. The correct match between item I and item II is:

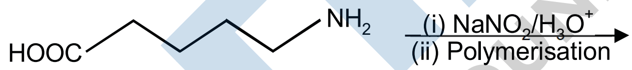
Item–I	**Item:II**
(A) Norethindrone	(P) Anti-biotic
(B) Ofloxacin	(Q) Anti-fertility
(C) Equanil	(R) Hypertension
	(S) Analgesics

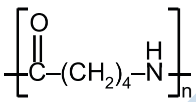
(1) (A)→(Q); (B)→(R); (C)→(S) (2*) (A)→(Q); (B)→(P); (C)→(R)

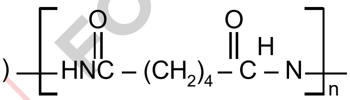
(3) (A)→(R); (B)→(P); (C)→(S) (4) (A)→(R); (B)→(P); (C)→(R)

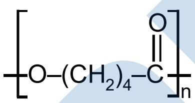
Sol. Fact based.

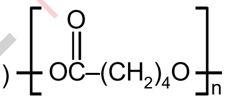
37. The polymer obtained from the following reactions is

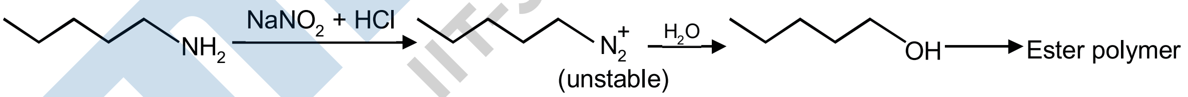


(1) 

(2) 

(3*) 

(4) 

Sol. 

38. NaH is an example of

(1) electron rich hydride (2) metallic hydride

(3*) saline hydride (4) molecular hydride

Sol. NaH is an example of saline hydride.

39. The element that usually does **not** show variable oxidation states is:

(1) Cu (2) Ti (3*) Sc (4) V

Sol. Sc^{3+} has noble gas configuration hence only +3 exists.

40. An example of solid sol is

(1) Paint (2*) Gem stones (3) Butter (4) Hair cream

Sol. An example of solid sol is gem stones.

41. The concentration of dissolved oxygen (DO) in cold water can go upto

(1) 14 ppm (2) 8 ppm (3*) 10 ppm (4) 16 ppm

Sol. In cold water, dissolved oxygen can reach a concentration upto 10 ppm

42. The correct statements among (a) to (d) regarding H_2 as a fuel are

(a) It produces less pollutants than petrol.

(b) A cylinder of compressed dihydrogen weighs ~30 times more than a petrol tank producing the same amount of energy.

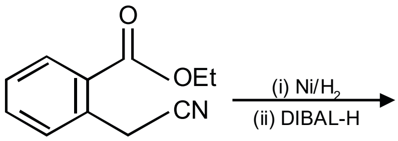
(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$

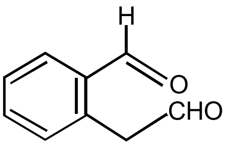
(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively.

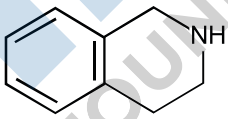
(1) (b) & (d) only (2) (a) & (c) only (3) (b), (c) & (d) only (4*) (a), (b) & (c) only

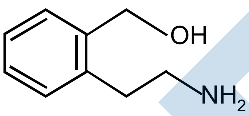
Sol. Fact based.

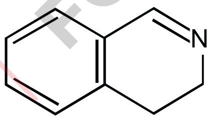
43. The major product of the following reaction is:



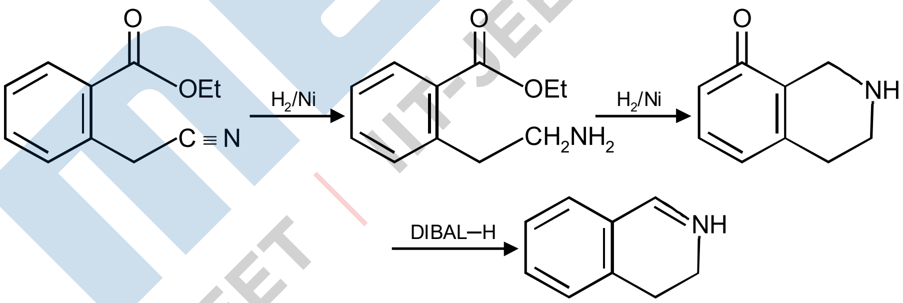
(1) 

(2) 

(3) 

(4*) 

Sol.



44. The freezing point of a diluted milk sample is found to be –0.2ºC, while it should have been –0.5ºC for pure milk. How much water has been added to pure milk to make the diluted sample?

(1) 1 cup of water to 2 cups of pure milk (2*) 3 cups of water to 2 cups of pure milk

(3) 1 cup of water to 3 cups of pure milk (4) 2 cups of water to 3 cups of pure milk

Sol. $0.5 \alpha \frac{1}{2}, 0.2 \alpha \frac{1}{x}$

Here $\frac{0.5}{0.2} = \frac{x}{2}; x = 5$, hence 3 cup

45. If a reaction follows the Arrhenius equation, the plot lnk vs 1/(RT) gives straight line with a gradient (–y) unit. The energy required to activate the reactant is

(1) y/R unit (2*) y unit (3) yR unit (4) –y unit

Sol. $\ell nk = -\dfrac{E_a}{RT} + \ell nA$

$\therefore$ Slope $= -E_a = -y$

46. A 10 mg effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet?

[Molar mass of $NaHCO_3$ = 84 g mol^{-1}]

(1*) 0.84 (2) 33.6 (3) 16.8 (4) 8.4

Sol. Let $NaHCO_3$ = x gm

Then, $H_2C_2O_4$ = (10 – x) gm

$\therefore n_{NaHCO_3} = \dfrac{x}{84}$

$2NaHCO_3 \rightarrow Na_2CO_3 + H_2O + CO_2$

$\therefore n_{CO_2} = \dfrac{x}{168}$

Total $CO_2 = \dfrac{x}{168} + \dfrac{10-x}{90} = \dfrac{0.2}{25}$

On solving 'x'

$\% = \dfrac{x}{10} \times 100 = 10x$

$n_{H_2C_2O_4} = \left(\dfrac{10-x}{90}\right)$

$H_2C_2O_4 \rightarrow H_2O + CO_2 + CO$

$\therefore n_{CO_2} = \left(\dfrac{10-x}{90}\right)$

47. The amphoteric hydroxide is

(1*) $Be(OH)_2$ (2) $Ca(OH)_2$ (3) $Mg(OH)_2$ (4) $Sr(OH)_2$

Sol. Be – O – H

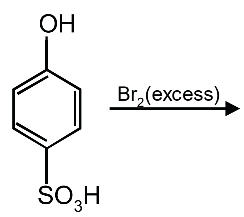
Both bond has same dissociation energy.

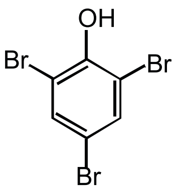
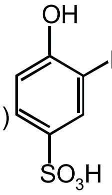
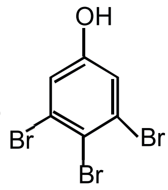
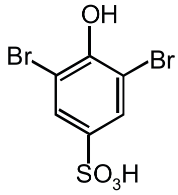
48. Peroxyacetyl nitrate (PAN), an eye irritant is produced by

(1) classical smog (2) acid rain (3) organic waste (4*) photochemical smog

Sol. Fact based.

49. The major product of the following reaction is:



(1*) 
(2) 
(3) 
(4) 

Sol. $-SO_3H$ will be replaced by Br this is called ipso effect.

50. An organic compound is estimated through Dumus method and was found to evolve 6 moles of CO_2, 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is:

(1) $C_{12}H_8N$ (2) $C_{12}H_8N_2$ (3*) $C_6H_8N_2$ (4) C_6H_8N

Sol. CO_2 = 6 mole, N_1 = 1 mole

C_{atom} = 6, N_{atom} = 2

Hence $C_6H_8N_2$

51. Consider the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2\ NH_3\ (g)$

The equilibrium constant of the above reaction is K_p. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $p_{NH_3} << p_{total}$ at equilibrium)

(1*) $\frac{3^{3/2}K_p^{1/2}p^2}{16}$ (2) $\frac{K_p^{1/2}p^2}{16}$ (3) $\frac{K_p^{1/2}p^2}{4}$ (4) $\frac{3^{3/2}K_p^{1/2}p^2}{4}$

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3$

equ^m x 3x P_1

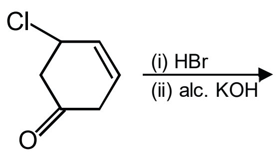
$$P_T = 4x\ K_p = \frac{P_1^2}{x \times 27 \times 3}$$

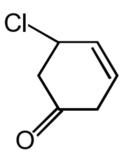
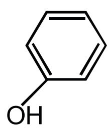
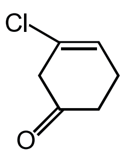
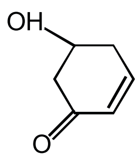
$$x = \left(\frac{P}{4}\right)$$

$$P_1 = \sqrt{24x^4K_p}$$

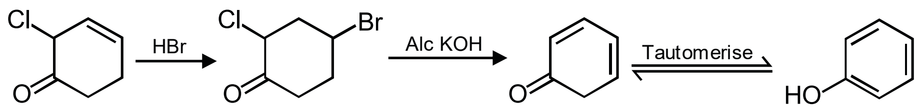
$$\sqrt{27}\,(K_p)^{1/2}\left(\frac{P_T}{4}\right)^2 = \frac{3^{3/2}K_p^{1/2}p^2}{16}$$

52. The major product of the following reaction is:



(1) 
(2*) 
(3) 
(4) 

Sol.



53. Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is:

(1*) $C_p \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (2) $2C_p \ln\left[\frac{(T_1+T_2)^{1/2}}{T_1T_2}\right]$ (3) $2C_p \ln\left[\frac{T_1+T_2}{4T_1T_2}\right]$ (4) $2C_p \ln\left[\frac{T_1+T_2}{2T_1T_2}\right]$

Sol. $\Delta S_{Total} = C\ell n\frac{(T_1+T_2)}{2T_1} + C_P \ell n\frac{(T_1+T_2)}{2T_2}$

54. Match the metals (Column I) with the coordination compound (s) / enzymes (s) (column II)

Column (I) Metals		Column (II) Coordination compound(s) /enzyme(s)
(A)	Co	(i) Wilkinson catalyst
(B)	Zn	(ii) Chlorophyll
(C)	Rh	(iii) Vitamin B_{12}
(D)	Mg	(iv) Carbonic anhydrase

(1*) (A)→(iii); (B)→(iv); (C)→(i) ; (D)→(ii) (2) (A)→(i); (B)→(ii); (C)→(iii) ; (D)→(iv)
(3) (A)→(ii); (B)→(i); (C)→(iv) ; (D)→(iii) (4) (A)→(iv); (B)→(iii); (C)→(i) ; (D)→(ii)

Sol. Co → Vitamin B_{12}

Zn → Carbonic anhydrase

Rh → Wilkinson catalyst

Mg → Chlorophyll

55. For the chemical reaction x ℓ y, the standard reaction Gibbs energy depends on temperatureT (in K) as

$$\Delta_r G^\circ \text{ (in kJ mol}^{-1}) = 120 - \frac{3}{8}T.$$

The major component of the reaction mixture at T is

(1) Y if T = 300 K (2) Y if T = 280 K (3) X if T = 350 K (4*) X if T = 315 K

Sol. $\Delta G^\circ = \left(120 - \frac{3}{8}T\right) = 0$

Then T = 320 K

Hence T > 320 K Y formed

T < 320 K X formed

56. Match the ores (Column A) with the metals (column B)

Column (A) Ores	Column (B) Metals
(I) Siderite	(a) Zinc
(II) Kaolinite	(b) Copper
(III) Malachite	(c) Iron

(IV) Calamine (d) Aluminium

(1) (I)→(a); (II)→(b); (III)→(c) ; (IV)→(d)

(2*) (I)→(c); (II)→(d); (III)→(b) ; (IV)→(a)

(3) (I)→(c); (II)→(d); (III)→(a) ; (IV)→(b)

(4) (I)→(b); (II)→(c); (III)→(d) ; (IV)→(a)

Sol. Siderite → Iron

Kaolinite → Aluminium

Malachite → Copper

Calamine → Zinc

57. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H–atom is suitable for this purpose?

[$R_H = 1 \times 10^5$ cm^{-1}, $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1}]

(1*) Paschen, $\infty \rightarrow 3$ (2) Paschen, $5 \rightarrow 3$ (3) Balmer, $\infty \rightarrow 2$ (4) Lyman, $\infty \rightarrow 1$

Sol. [i] 900 nm = 9000 Å

It is in far infrared region hence paschen.

58. The chloride that CANNOT get hydrolysed is:

(1) $PbCl_4$ (2*) CCl_4 (3) $SnCl_4$ (4) $SiCl_4$

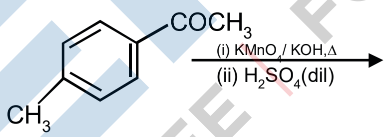
Sol. Central atom has no vacant orbital.

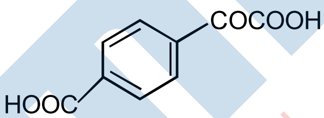
59. The correct order of the atomic radii of C, Cs, Al and S is

(1*) C < S < Al < Cs (2) S < C < Cs < Al (3) S < C < Al < Cs (4) C < S < Cs < Al

Sol. On moving down size increases.

60. The major product of the following reaction is



(1)


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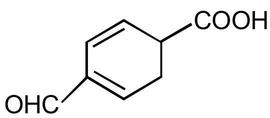
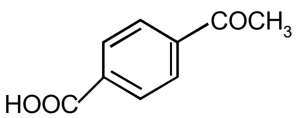


(3*)


(4)


Sol. It is the case of side chain oxidation.

PART-C-PHYSICS

61. A body is projected at t = 0 with a velocity $10ms^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = $10ms^{-2}$, the value of R is

(1) 10.3 m (2*) 2.8 m (3) 2.5 m (4) 5.1 m

Sol. at t = 1

$u_x = 5, u_y = 5\sqrt{3}$

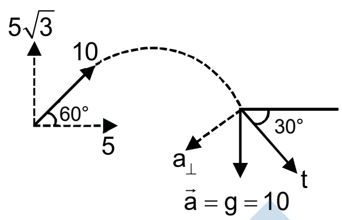
$V_y = 5\sqrt{3} - 10$; $V_x = 5$

$\tan\theta = -\left(2-\sqrt{3}\right) \Rightarrow \theta = -30°$

$R = \frac{v^2}{a_\perp} = \frac{10^2}{(10\cos 30°)}$

$= \frac{10}{\sqrt{3}} \times 2 = \frac{20}{\sqrt{3}} m$

$\frac{5^2 + \left(10-5\sqrt{3}\right)^2}{10\cos\theta} = \frac{200-100\sqrt{3}}{10 \times 0.965} = 2.8 \text{ m}$



62. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be

(hc = 12500 eV-Å)

(1) $25a_0$ (2) $9a_0$ (3*) $16a_0$ (4) $4a_0$

Sol. Energy supplied

$E = \frac{12400}{900} = 12.65 \text{ eV}$

$\therefore E_n - E_1 = 12.65$

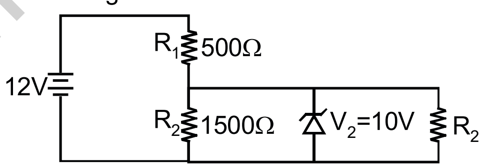
$\Rightarrow (13.6)\left(1-\frac{1}{n^2}\right) = 12.65$

$\Rightarrow n^2 \approx 14.3$

$\Rightarrow n \approx 4$

$R \propto n^2$

63. In the given circuit the current through Zener Diode is close to



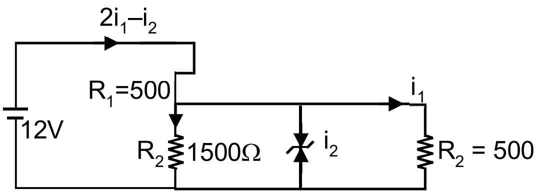
(1*) 0.0 mA (2) 6.7 mA (3) 4.0 mA (4) 6.0 mA

Sol. 12 – 500(2i1 + i2) – 10 = 0

$\Rightarrow 2i_1 + i_2 = \frac{2}{500} = \frac{1}{500}$

< i_1 when zenor has break down.

So, i_2 = 0.



64. There are two long co-axial solenoids of same length *l*. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self nductance of the inner-coil is

(1*) $\frac{n_1}{n_2}$ (2) $\frac{n_2}{n_1}\cdot\frac{r_1}{r_2}$ (3) $\frac{n_2}{n_1}\cdot\frac{r_2^2}{r_1^2}$ (4) $\frac{n_2}{n_1}$

Sol. $\frac{M}{L}=\frac{\mu_0 n n_1 n_1^2 \ell}{\mu_0 \pi n_1^2 r_1^2 \ell}$

$=\frac{n_2}{n_1}$

65. A particle undergoing simple harmonic motion has time dependent displacement give by

$x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be

(1) $\frac{1}{9}$ (2) 1 (3) 2 (4*) 3

Sol. $K=\frac{1}{2}mv^2$; $U=\frac{1}{2}kx^2=\frac{1}{2}m^2x^2$

$\therefore \frac{k}{U}=\frac{v^2}{\omega^2x^2}=\left(\frac{\cos(wt)}{\sin(wt)}\right)^2$

$=\cot^2\left(\frac{\pi}{90}\times 210\right)$

$=\cot^2\left(2\pi+\frac{\pi}{3}\right)$

$=\left(\frac{1}{\sqrt{3}}\right)^2=\frac{1}{3},$

66. A satellite is revolving in a circular orbit at a height h from the earth surface, such that h <<R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is

(1) $\sqrt{2gR}$ (2) $\sqrt{gR}$ (3) $\sqrt{\frac{gR}{2}}$ (4*) $\sqrt{gR}\left(\sqrt{2}-1\right)$

Sol. $\Delta V = V_f - V_i$

$=\sqrt{\frac{2gMe}{R_e}}-\sqrt{\frac{gMe}{R_e}}$

$=\left(\sqrt{2}-1\right)\sqrt{gR_e}$

67. The force of interaction between two atoms is given by F = $\alpha\beta$ exp; $\left(-\frac{x^2}{\alpha kt}\right)$ where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is:

(1) $M^0L^2T^{-4}$ (2*) M^2LT^{-4} (3) MLT^{-2} (4) $M^2L^2T^{-2}$

Sol. Power of e should be dimensionless.

So, $[\lambda] = (\alpha Tk)$

$\Rightarrow L^2 = [\alpha]\,(ML^2\,T^2)$

$\Rightarrow (\alpha) = (M^{-1}\,T^2)$

$\Rightarrow E = \frac{1}{2}KT$

$\Rightarrow (ML^2T^{-2})$; $(E) = [KT]$

$\Rightarrow (\alpha\beta) = (F)$

$\Rightarrow (M^{-1}\,T^2)\,(\beta) = (MLT^{-2})$

68. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$ th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to:

(1) 0.74 (2*) 0.85 (3) 0.94 (4) 0.80

Sol. $\Delta x = \frac{\lambda}{8}$

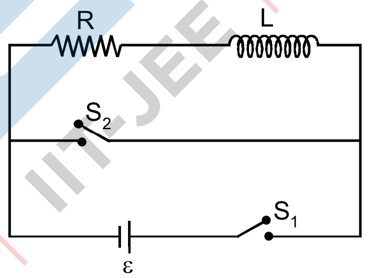
Phase $|\Delta P| = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

$\therefore \quad I_{res} = I + I + 2I\cos\left(\frac{R}{4}\right)$

$= 2I\left(1 + \frac{1}{\sqrt{2}}\right) = 2I \times 1.7$

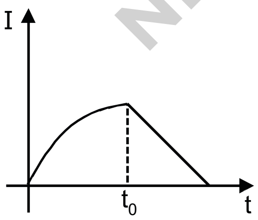
$\therefore \quad \frac{I_{res}}{I_{man}} = \frac{2I \times 1.7}{4I} = 0.85$

69. In the circuit shown:

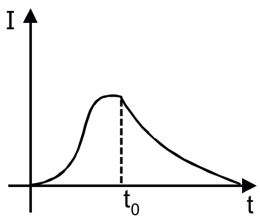


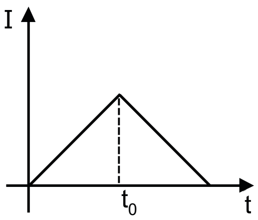
The switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by:

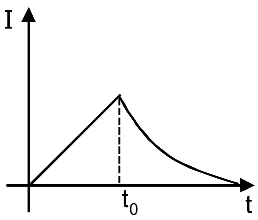
(1)

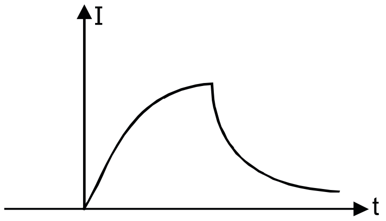


(2*)



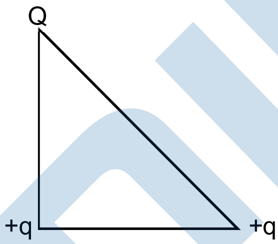
(3)


(4)


Sol.


Above is the correct graph for growth and decay of current.

70. Three charges Q, +q and +q are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is is zero, if the value of Q is

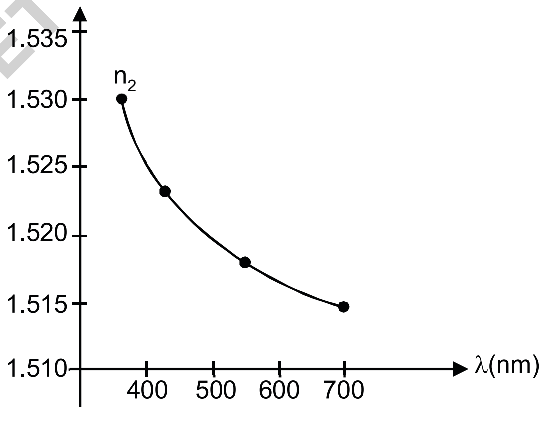


(1) +q (2*) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (3) $\frac{-q}{1+\sqrt{2}}$ (4) –q

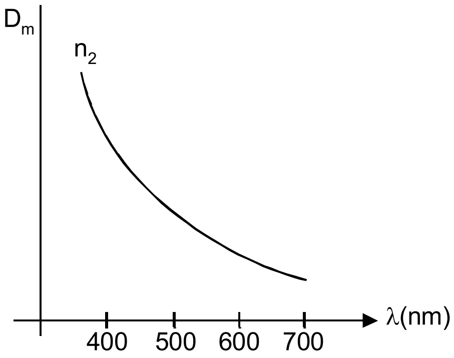
Sol. $U_{Total} = \frac{kQq}{a} + \frac{kq^2}{a} + \frac{kQq}{a\sqrt{2}} = 0$

$\Rightarrow Q = \frac{-q}{\left(1+\frac{1}{\sqrt{2}}\right)}$

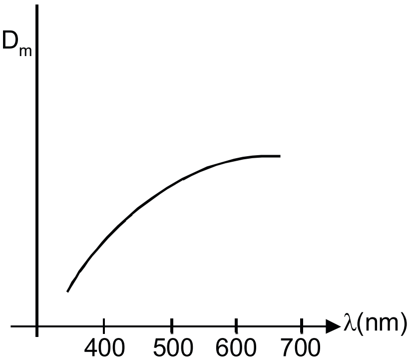
71. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



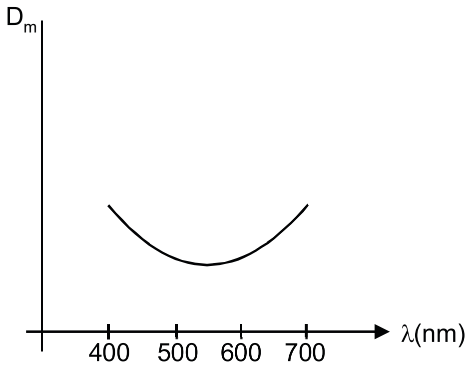
(1*)



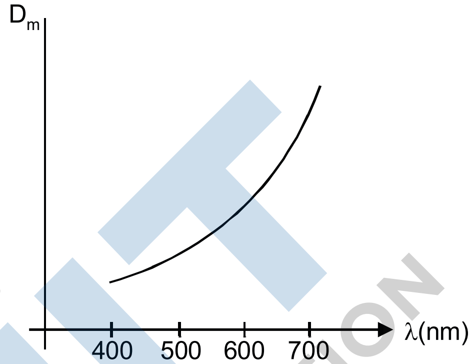
(2)



(3)



(4)

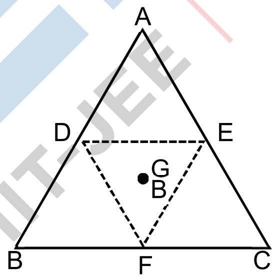


Sol. Prism formula

$D_m = S_m = (n - 1)$ A (for thin prism)

So, answer is 1.

72. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then



(1*) $I = \frac{15}{16} I_0$ (2) $I = \frac{3}{4} I_0$ (3) $I = \frac{9}{16} I_0$ (4) $I = \frac{I_0}{4}$

Sol. $I \propto m\ell^2$ (let σ = mass present area)

$\therefore I_1 \propto \ell 4$ ...(1)

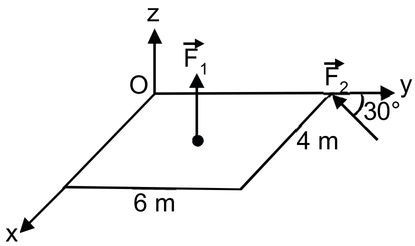
and $I_2 \propto \left(\frac{\ell}{2}\right)^4$ .....(2)

So, $I_2 = \frac{I}{16}$

Moment of inertia of remaining sheet $= I - \frac{I}{16}$

$= \frac{15I}{16}$

73. A slab is subjective to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z-axis at the point $\left(2\vec{i}+3\vec{j}\right)$. The moment of these forces about point O will be:



(1*) $\left(3\vec{i}-2\vec{j}+3\hat{k}\right)F$ (2) $\left(3\vec{i}-2\vec{j}-3\hat{k}\right)F$ (3) $\left(3\vec{i}+2\vec{j}-3\hat{k}\right)F$ (4) $\left(3\vec{i}+2\vec{j}+3\hat{k}\right)F$

Sol. $\vec{\tau}_0 = \vec{\tau}_1 + \vec{\tau}_2$

$= \left(6\hat{i}\right)\times\left(-\frac{F}{2}\hat{i}+\frac{F\sqrt{3}}{2}\hat{j}\right)+\left(2\hat{i}+3\hat{j}\right)\times\left(F\hat{k}\right)$

$= -3F\,\hat{k}\left(-2F\hat{j}+3F\hat{i}\right)$

$= F\left(3\hat{i}-2\hat{j}-3\hat{k}\right)$

74. Ice at –20ºC is added to 50g of water at 40ºC. When the temperature of the mixture reaches 0ºC, it is found that 20g of ice is still unmelted. The amount of ice added to the water was close to
(Specific heat of water = 4.2 J/g/6C, Specific heat of ice = 2.1 J/g/ºC, Heat of fusion of water at 0ºC = 334 J/g)

(1) 50 g (2) 100 g (3) 60 g (4*) 40 g

Sol. = 50 × 1 × 40

⇒ 90 m – 1600 = 2000

⇒ 90 m = 3600

⇒ m = 40 gm

75. A particle is moving along a circular path with a constant speed of $10ms^{-1}$. What is the magnitude of the of the change in velocity of the particle, when it moves through an angle of 60º around the centre of the circle?

(1) $10\sqrt{3}m/s$ (2) zero (3) $10\sqrt{2}m/s$ (4*) 10 m/s

Sol. $\Delta\vec{v} = 2v\sin\left(\frac{\theta}{2}\right)$

= 2 × 10 × sin(30°)

= 10 m/s

76. An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$.
Here t is in seconds. The sideband frequencies (in kHz) are, [given π = 22/7]

(1) 1785 and 1715 (2) 178.5 and 171.5 (3*) 89.25 and 85.75 (4) 892.5 and 857.5

Sol. $f_{so} = f_c \pm f_m$

$= \frac{\omega_c \pm \omega_m}{2\pi}$

$= \frac{(5.5 \pm 0.22)\times 10^5}{2\times\frac{22}{7}}$

= 89.25, 85.75

77. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. If the object moves away from the lens at a speed of 5m/s. The speed and direction of the image will be:

(1) 2.26×10^{-3} m/s away from the lens

(2) 0.92×10^{-3} m/s away from the lens

(3) 3.22×10^{-3} m/s towards the lens

(4*) 1.16×10^{-3} m/s towards the lens

Sol. $\frac{1}{V} - \frac{1}{-20} = \frac{1}{30}$

$\Rightarrow \frac{1}{V} = \frac{1}{0.30} - \frac{1}{20} = +\frac{200-3}{60} = \frac{197}{60}$

$\Rightarrow -\frac{dv}{dt\ V^2} + \frac{du}{dt\ u^2} = 0$

$\Rightarrow \frac{dv}{dt} = \frac{du}{dt\ u^2} = 0$

$\Rightarrow \frac{dV}{dt} = \frac{V^2}{u^2}\frac{du}{dt} = \left(\frac{3}{197}\right)^2 \times (-5) = -0.00113\ m/s$

78. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is

(1) 15 RT (2*) 12 RT (3) 4 RT (4) 20 RT

Sol. $U_{total} = U_{O_2} + U_{Ar}$

$= \frac{3\times 5\times RT}{2} + \frac{5\times 3\times RT}{2}$

= 15 RT

79. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be:

(1) 60 W (2*) 240 W (3) 120 W (4) 30 W

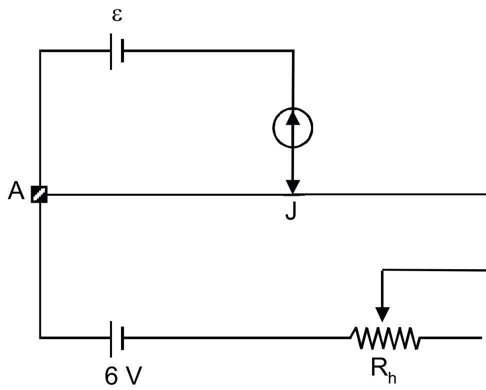
Sol. Assuming constant voltage supply

$P = 60 = \frac{V^2}{R_1 + R_2}$ (1)

And $P' = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{2V^2}{R_1} = 4P = 4\times 60$

= 240 W

80. The resistance of the meter bridge AB in given figure is 4Ω. With a cell of emf ε = 0.5 V and rheostate resistance R_h = 2Ω the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for R_h = 6Ω. The emf ε_2 is:



(1) 0.4 (2*) 0.3 (3) 0.6 (4) 0.5

Sol. $0.5 = \frac{6}{(2+\lambda L)}\lambda x$ ....(1)

$E_2 = \frac{6}{(6+\lambda L)}\lambda x$ ......(2)

So dividing equation (1) and (2)

$\frac{E_2}{0.5} = \frac{2+4}{6+4} = \frac{3}{5}$

$\Rightarrow$ E_2 = 0.3 Volt.

81. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by:

(1) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (2*) $\left(\sqrt{n}, \sqrt{n}\right)$ (3) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$ (4) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

Sol. $\frac{E_i}{B_i} = C$ .....(1)

$\frac{E_f}{B_f} = \frac{c}{n}$ .....(2)

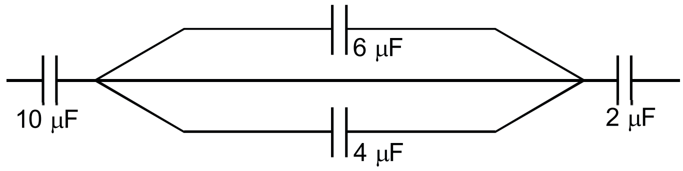
$\Rightarrow \frac{E_i B_f}{E_f B_i} = \frac{1}{n}$

$\Rightarrow \frac{E_i}{E_f} = \frac{1}{n}\frac{B_i}{B_f}$

$\left(\because\ n = n = \frac{1}{\sqrt{u_o e_r}}\right)$

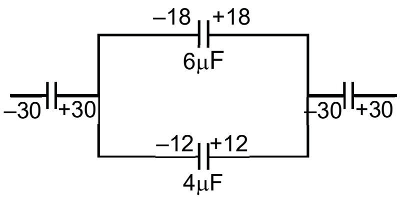
$\frac{1}{\sqrt{n}}; \sqrt{n}$

82. In the figure shown below, the charge on the left plate of the 10μF capacitor is –20 μC. The charge on the right plate of the 6μF capacitor is:



(1) –12μC (2) +12 μC (3) –18μC (4*) +18μC

Sol.



$4V + 6V = 30$

$\Rightarrow \quad V = 3$

83. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^{T} = constant, then x is:

(1) 3/5 (2*) 2/5 (3) 2/3 (4) 5/3

Sol. Equation of adiabatic process

$TV^{2/f}$ = constant

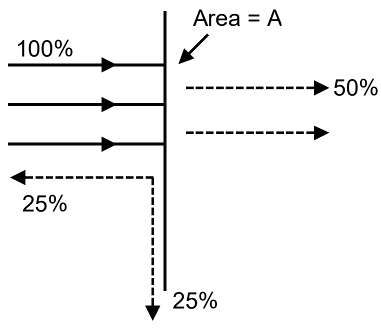
$\therefore \quad \frac{2}{f} = \frac{2}{5} = x$

84. A liquid density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be:

(1) $\frac{1}{4}\rho v^2$ (2*) $\frac{3}{4}\rho v^2$ (3) $\frac{1}{2}\rho v^2$ (4) ρv^2

Sol. $F = \frac{1}{4} \times \rho A v^2 + \frac{1}{4} 2\rho a v^2$

$\Rightarrow \quad \frac{F}{A} = \frac{3}{4}\rho A v^2 = \rho$



85. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to:

(Speed of light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} J.s, Mass of electron = 9.1×10^{-32} kg)

(1) 1.1×10^6 m/s (2) 1.7×10^6 m/s (3) 1.8×10^6 m/s (4*) 1.45×10^6 m/s

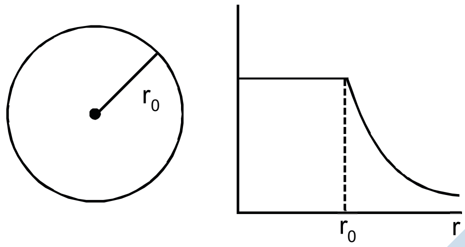
Sol. $\lambda_e = \lambda_{photon} \times 10^{-3}$

$\Rightarrow \quad v = \dfrac{h\nu}{mC \times 10^{-3}}$

$= \dfrac{6.62 \times 10^{-34} \times 6 \times 10^{14}}{9.1 \times 10^{-31} \times 3 \times 10^{8} \times 10^{-3}}$

$= 1.45 \times 10^6$

86. The given graph shows variation (with distance r from centre) of



(1) Electric field of a uniformly charged sphere
(2*) Potential of a uniformly charged spherical shell
(3) Potential of a uniformly charged sphere
(4) Electric field of a uniformly charged spherical shell

Sol. The potential inside a uniformly charged shell is constant, while it decrease hyperbolically outside.

87. Equation of a travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin (450 t – 9x) where distance and time are measured in SI units. The tension in the string is:

(1) 10 N (2) 7.5 N (3*) 12.5 N (4) 5 N

Sol. $y = 0.03\left[450\left(t - \dfrac{9x}{450}\right)\right]$

So, $v = \dfrac{450}{9} = 50 \text{ m/s}$

Also, $v = \sqrt{\dfrac{T}{\lambda}}$

$\Rightarrow \quad T = 2500 \times 5 \times 10^{-3} = 12.5$ N

88. In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of a path if a magnetic field 100 mT is then applied. [Charge on electron = 1.6×10^{-19}C Mass of the electron = 9.1×10^{-31} kg]

(1) 7.5×10^{-3} m (2) 7.5×10^{-2} m (3) 7.5 m (4*) 7.5×10^{-4} m

Sol. $k_e = \dfrac{p^2}{2M_e} = 500 \text{ e}$ ....(i)

& $R = \dfrac{p}{eB} = \dfrac{\sqrt{1000 m_e e}}{eB} = \dfrac{\sqrt{1010 \times 9.1 \times 10^{-31}}}{1.6 \times 10^{-19}}$

$= 100 \times 7.541 \times 10^{-6}$

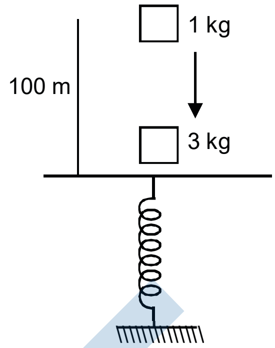
89. A body of mass 1 kg falls freely from a height of 100 cm, on a platform of mass 3kg which is mounted on a spring having spring constant k = 1.25 × 10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = $10ms^{-2}$, the value of x will be close to

(1) 40 cm (2*) 4 cm (3) 80 cm (4) 8 cm

Sol. $v = \sqrt{2} \times 10 \times 100$

$= 20\sqrt{5}$

COTME →]



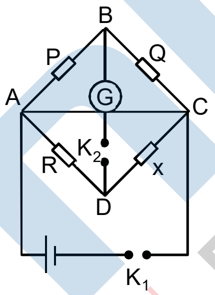
$$\frac{1}{2} \times 4 \times \left(5\sqrt{5}\right)^2 + \frac{1}{2} 1.25 \times 10^6 \left(\frac{30}{k}\right)^2$$

$$= -4 \times 10 \times \left(n - \frac{30}{k}\right) + \frac{1}{2} kn^2$$

$$\Rightarrow \quad 250 + \frac{900}{2 \times 1.25 \times 10^6} = -40x + \frac{1200}{k} + \frac{1}{2} kx^2$$

$\Rightarrow$ x = 2cm

90. In a Wheatstone bridge (as shown in figure) Resistances P and Q are approximately equal. When R = 400Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is 405 Ω. The value of X is close to:



(1) 401.5 ohm (2) 404.5 ohm (3) 403.5 ohm (4*) 402.5 ohm

Sol. $\frac{P}{Q} = \frac{400}{S}$ ....(1)

and $\frac{Q}{P} = \frac{405}{5}$ ....(2)

Solving $S^2 = 400 \times 405$

$\Rightarrow$ S = 402.5 Ω